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EUROPEAN ORGANIZATION FOR PARTICLE PHYSICS

CERN/PPE/93-157
26 August 1993

Updated Parameters of the Z^0 Resonance from Combined Preliminary Data of the LEP Experiments*

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and
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Conference contribution
prepared for the Europhysics Conference on High Energy Physics, Marseille, July 22–28 1993
and the 1993 International Symposium on Lepton-Photon Interactions at High Energies,
Cornell, August 10–15.

Abstract

The data recorded by the four LEP experiments until the end of 1992 correspond to approximately $5 \cdot 10^6$ Z^0 decays into hadrons and charged leptons. This note presents a combination of published and preliminary electroweak results from the four LEP collaborations which were prepared for the Europhysics Conference on High Energy Physics, Marseille, July 22–28 1993 and the 1993 International Symposium on Lepton-Photon Interactions at High Energies, Cornell, August 10–15. We present averages of electroweak LEP results from the measurement of hadronic and leptonic cross sections, the leptonic forward-backward asymmetries, the τ polarization asymmetries, the $b\bar{b}$ and $c\bar{c}$ partial widths and forward-backward asymmetries and the $q\bar{q}$ charge asymmetry. To demonstrate the sensitivity of the data, the combined set of electroweak measurements is used to constrain the parameters of the Standard Model.

*The LEP Collaborations each take responsibility for the preliminary data of their own experiments. The choice of parameters and the averaging procedures are under the responsibility of the LEP Electroweak Working Group.

1 Introduction

The four LEP experiments present updated parameters of the Z^0 resonance derived from published data and new preliminary results. Most of these preliminary results are contributions to the 1993 summer conferences at Marseille and Cornell. The electroweak parameters of the four LEP experiments have been combined, taking account of errors which are correlated among the experiments, by the LEP Electroweak Working Group[1].

Previously a procedure was proposed for combining the results from the LEP experiments for hadronic and leptonic cross sections and the leptonic forward-backward asymmetries [1, 2]. This procedure has introduced certain standards in the presentation of results among the four LEP experiments and is now well established. In section 2 we apply the procedure to the new data available in order to provide improved LEP averages.

With the increased statistics, and the new silicon microvertex detectors, also other electroweak measurements provide significant constraints on Standard Model parameters. This note therefore also summarizes the procedures available so far for combining the LEP results for the τ polarization asymmetries (section 3), the $b\overline{b}$ and $c\overline{c}$ partial widths and forward-backward asymmetries (section 4) and the $q\overline{q}$ charge asymmetry (section 5). The latter procedures are still under development and are likely to be refined for future data.

The data collected at LEP have also contributed a variety of further tests of the electroweak theory, such as the study of single photons attributed to the process $e^+e^- \to \nu\overline{\nu}\gamma$ and the determination of the final state photon yield in hadronic events. These measurements represent an important confirmation of the Standard Model. With their present accuracy, however, they do not contribute to the constraints on Standard Model parameters presented in this note.

In section 6 we study the sensitivity and consistency of the individual measurements. These are combined in section 7 in order to constrain the parameters of the Standard Model.

2 Z^0 Lineshape and Lepton Forward-Backward Asymmetries

Details of the individual analyses can be found in [3, 4, 5, 6]. The results presented here are based on the results from two energy scans in 1990 and 1991 with centre-of-mass energies in a range $\sqrt{s} = M_Z \pm 3$ GeV, and the high statistics data collected at the Z^0 peak in 1992. The total statistics and the systematic errors of the individual LEP collaborations are given in Tables 1 and 2.

An important aspect of the lineshape analysis is a precise knowledge of the LEP centre-of-mass energies. The treatment of the LEP centre-of-mass energies by the four LEP experiments is based on [7].

For the averaging of results the LEP experiments provide a standard set of 9 parameters describing the information contained in hadronic and leptonic cross-sections and leptonic forward-backward asymmetries:

[1]The present members of the LEP Electroweak Working Group are: A. Blondel, I. Brock, R. Clare, A. Falvard, S. Ganguli, M. Mannelli, M. Martinez, K. Mönig, G. Myatt, A. Olshevsky, G. Quast, M. Pepe-Altarelli, P. Renton, E. Ros, D. Schaile, D. Schlatter, R. Tenchini, I. TenHave, P.S. Wells.

- The mass of the Z^0, M_Z, and the total width, Γ_Z, where the definition is based on the Breit Wigner denominator $(s - M_Z^2 + is\Gamma_Z/M_Z)$.
- The hadronic pole cross section:

$$\sigma_h^0 \equiv \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{had}}{\Gamma_Z^2}.$$

Here Γ_{ee} and Γ_{had} are the partial widths of the Z^0 for decays into electrons and hadrons.

- The ratios:

$$R_e \equiv \Gamma_{had}/\Gamma_{ee} \qquad R_\mu \equiv \Gamma_{had}/\Gamma_{\mu\mu} \qquad R_\tau \equiv \Gamma_{had}/\Gamma_{\tau\tau}\,.$$

Here $\Gamma_{\mu\mu}$ and $\Gamma_{\tau\tau}$ are the partial widths of the Z^0 for the decays $Z^0 \to \mu^+\mu^-$ and $Z^0 \to \tau^+\tau^-$.

- The pole asymmetries, $A_{FB}^{0,e}$, $A_{FB}^{0,\mu}$ and $A_{FB}^{0,\tau}$ for the processes $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$. In terms of the effective vector and axial-vector neutral current couplings of fermions, g_{V_f} and g_{A_f}, the pole asymmetries can be expressed as[2]:

$$A_{FB}^{0,f} \equiv \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \qquad (1)$$

with

$$\mathcal{A}_f = \frac{2g_{V_f}g_{A_f}}{g_{V_f}^2 + g_{A_f}^2}. \qquad (2)$$

The four sets of 9 parameters provided by the LEP experiments from fits to the data taken until the end of 1992 are presented in Table 3.

In a previous paper [1] it was shown that the values of electroweak parameters derived by the LEP experiments from the data taken in 1989 and 1990 could be combined by taking a simple weighted mean of the fit variables. For the calculation of derived quantities an average correlation matrix, or the matrix of any one of the experiments, could be used with sufficient precision. Errors which were common to the experiments arose from the absolute LEP energy scale, the relative energies of the different scan points and the theoretical uncertainty in the Bhabha cross section used in the luminosity determinations. The theoretical uncertainty in the t-channel subtraction in the e^+e^- channel gave a negligible contribution to the common errors.

A slightly more elaborate procedure, as described below, was used for averaging the LEP results available for the XXVI International Conference on High Energy Physics Dallas, Texas, USA, August 5-12 1992 [2] and will be used here.

Each of the experiments provided a correlation matrix for their parameters, and also a matrix including only the uncertainties introduced by the LEP energy calibrations. In addition, the theoretical uncertainty for the calculation of the small angle Bhabha cross-section, which is assumed to be fully correlated among the experiments, has been taken into account. The information above is used to construct the full covariance matrix $\mathcal{V}$ of the input parameters (see Tables 4, 5 and 6). Then a combined parameter set is obtained by minimizing $\chi^2 = \Delta^T\mathcal{V}^{-1}\Delta$, where Δ denotes the vector of residuals of the combined parameter set to the results of the individual experiments. The combined parameter set and its correlation matrix are given in Tables 7 and 8.

The parameters given in the preceding tables make no assumption on lepton universality. Lepton universality means that the gauge couplings of the three known lepton doublets are assumed to be

[2] Effects coming from photon exchange, as well as real and imaginary parts of the photon vacuum polarization are not included in the definition of $A_{FB}^{0,f}$, but are accounted for explicitly in the fitting formulae used by the experiments.

equal. This assumption can be used to reduce the set of 9 parameters given above to a set of five parameters. Due to mass corrections to $\Gamma_{\tau\tau}$ we expect, however, a small (0.2%) difference between the values for R_e and R_μ and the value for R_τ. The procedure as to how to take into account these mass terms, when specifying a single partial width, $\Gamma_{\ell\ell}$, for the decay of the Z^0 into leptons is not unambiguous and has to be defined. At present three definitions are used:

i) $\Gamma_{\ell\ell}$ is defined as the partial Z^0 width for the decay into a pair of massless charged leptons. In terms of effective couplings, $\Gamma_{\ell\ell}$ can be written as:

$$\Gamma_{\ell\ell} = \frac{G_F M_Z^3}{6\pi\sqrt{2}}(g_{V_\ell}^2 + g_{A_\ell}^2)(1+\delta_{QED}), \tag{3}$$

where $\delta_{QED} = 3\alpha/(4\pi)$ accounts for final state photonic corrections.

ii) $\Gamma_{\ell\ell}$ is defined as the unweighted average:

$$\Gamma_{\ell\ell} = \frac{1}{3}(\Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau}).$$

iii) $\Gamma_{\ell\ell}$ is defined as the weighted average of Γ_{ee}, $\Gamma_{\mu\mu}$ and $\Gamma_{\tau\tau}$.

The differences among the definitions are negligible for any single experiment but amount to 30% of the error of the LEP average for $R_\ell = \Gamma_{had}/\Gamma_{\ell\ell}$. All numbers involving $\Gamma_{\ell\ell}$ in this report are based on the first definition[3].

The data are consistent with lepton universality. Based on this assumption Tables 9 and 10 provide the five parameters M_Z, Γ_Z, σ_h^0, R_ℓ and $A_{FB}^{0,\ell}$ and the corresponding correlation matrix. These were obtained from a parameter transformation of the LEP average for the 9 parameter fit (Tables 7 and 8), which has an unambiguous treatment of mass terms. For comparison we also give the results of the individual experiments from the 5 parameter fit in Table 11. Figure 1 shows, for each leptonic species and for the combination assuming lepton universality, the resulting one standard deviation contours in the R_ℓ-$A_{FB}^{0,\ell}$ plane.

The parameter R_ℓ can be used to determine the strong coupling constant α_s. For $M_Z = 91.187$ GeV, $M_t = 150$ GeV and $M_H = 300$ GeV we obtain from $R_\ell = 20.763 \pm 0.049$ alone:

$$\alpha_s(M_Z^2) = 0.123 \pm 0.007.$$

The error quoted accounts for experimental uncertainties only. The result for α_s has been derived based on the most up to date Standard Model calculations [8][4]. A more precise value of $\alpha_s(M_Z^2)$ will be given in the context of a global fit to all electroweak precision measurements in section 7.

The parameters M_Z, Γ_Z, σ_h^0, R_ℓ and $A_{FB}^{0,\ell}$ are convenient for fitting and averaging since they have minimal correlations amongst them. However a number of other parameters, which can be derived from the previous set, are of physical importance. Table 12 gives a number of commonly used parameters derived from the results of the 9-parameter (Tables 7 and 8) and the 5-parameter fit (Tables 9 and 10). Among these parameters we quote $\sin^2\theta_{eff}^{lept}$ which is defined as:

$$\sin^2\theta_{eff}^{lept} \equiv \frac{1}{4}(1 - g_{V_\ell}/g_{A_\ell}). \tag{4}$$

[3]In [1] definition ii) was used. We give preference to definition i) here, as it simplifies the calculation of derived quantities.

[4]Note, that simple formulae which parametrize the α_s dependence of R_ℓ as a power series in α_s/π do not account for $\mathcal{O}(\alpha\alpha_s)$ corrections and therefore have to be carefully normalized to these calculations in order to reproduce the results.

As an indication of the power of these new data we use them to derive the number of light neutrino species. Using the results of Tables 9 and 10 we find:

$$\Gamma_{\rm inv}/\Gamma_{\ell\ell} \quad = \quad 5.936 \pm 0.054\,.$$

Taking the Standard Model value for the ratio of the partial widths to neutrinos and charged leptons:

$$\Gamma_{\nu}/\Gamma_{l} \quad = \quad 1.992 \pm 0.003\,,$$

where the central value is evaluated for $M_{\rm Z} = 91.187$ GeV, $M_{\rm t} = 150$ GeV, $M_{\rm H} = 300$ GeV and $\alpha_s(M_{\rm Z}^2) = 0.123$ and the error quoted accounts for a variation of $M_{\rm t}$ in the range $100 < M_{\rm t}({\rm GeV}) < 200$ and a variation of $M_{\rm H}$ in the range $60 < M_{\rm H}({\rm GeV}) < 1000$, we find:

$$N_{\nu} \quad = \quad 2.980 \pm 0.027\,.$$

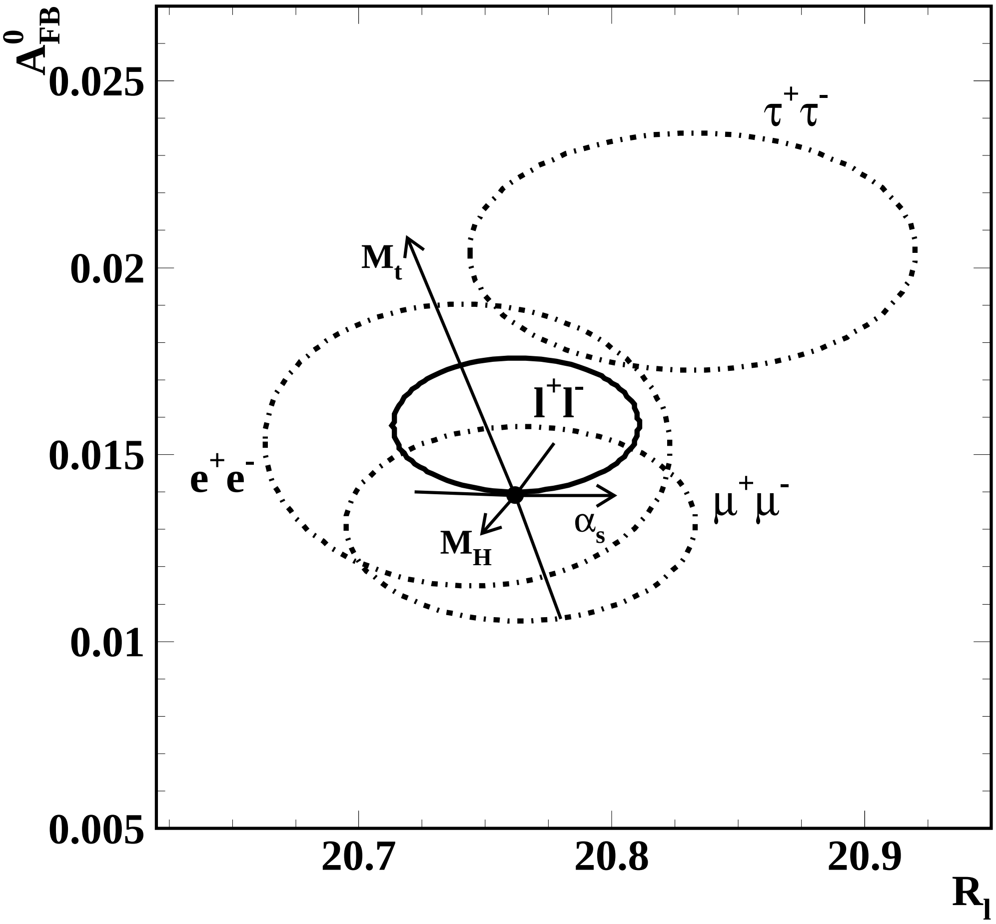

Figure 1: One standard deviation contours (39% probability) in the R_ℓ-$A_{\rm FB}^{0,\ell}$ plane. Also shown as dotted symbol is the Standard Model prediction for $M_{\rm Z} = 91.187$ GeV, $M_{\rm t} = 150$ GeV, $M_{\rm H} = 300$ GeV, $\alpha_s = 0.123$. The lines with arrows correspond to the variation of the Standard Model prediction when $M_{\rm t}$, $M_{\rm H}$ or α_s are varied in the intervals $50 < M_{\rm t}({\rm GeV}) < 250$, $60 < M_{\rm H}({\rm GeV}) < 1000$ and $\alpha_s(M_{\rm Z}^2) = 0.123 \pm 0.006$, respectively. The arrows point in the direction of increasing values for $M_{\rm t}$, $M_{\rm H}$ and α_s.

		ALEPH	DELPHI	L3	OPAL	LEP
$q\bar{q}$	'90-'91	451	365	423	454	1693
	'92	686	695	677	733	2791
	total	1137	1060	1100	1187	4484
$\ell^+\ell^-$	'90-'91	55	37	40	58	190
	'92	82	76	58	88	304
	total	137	113	98	146	494

Table 1: The LEP statistics in units of 10^3 events used for the analysis of the Z^0 line shape and lepton forward-backward asymmetries.

	ALEPH		DELPHI		L3		OPAL	
	'91	'92 prel.	'91	'92 prel.	'91	'92 prel.	'91	'92
$\mathcal{L}^{\text{exp.}\,(a)}$	0.45%	0.35%$^{(b)}$ 0.15%$^{(c)}$	0.5 %	0.6 %	0.5 %	0.5 %	0.60%	0.41%
σ_{had}	0.2 %	0.17%	0.2 %	0.28%	0.15%	0.14%	0.20%	0.20%
σ_{e}	0.4 %	0.4 %	0.5 %	0.6 %	0.4 %	0.4 %	0.45%	0.22%
σ_μ	0.5 %	0.5 %	0.5 %	0.5 %	0.5 %	0.5 %	0.25%	0.19%
σ_τ	0.6%	0.5%	0.75%	0.75%	0.7 %	0.7 %	0.76%	0.44%
$A_{\text{FB e}}$	$^{(d)}$	$^{(d)}$	0.002	0.004	0.0045	0.002	0.003	0.002
$A_{\text{FB}\,\mu}$	$^{(d)}$	$^{(d)}$	0.003	0.002	0.003	0.003	0.003	0.001
$A_{\text{FB}\,\tau}$	$^{(d)}$	$^{(d)}$	0.002	0.004	0.006	0.003	0.003	0.002

Table 2: The experimental systematic errors for the analysis of the Z^0 line shape and lepton forward-backward asymmetries. The errors quoted do not include the common uncertainty due to the LEP energy calibration. For the treatment of correlations between the errors for different years we refer to [3, 4, 5, 6].

$^{(a)}$Only the experimental error including the statistics of small angle Bhabha events is quoted. In addition, there is a theoretical error for the calculation of the small angle Bhabha cross section. The experiments have a different acceptance for small angle Bhabha scattering and quote a different value for this theoretical error: 0.25% for ALEPH and L3; 0.3% for DELPHI and OPAL.

$^{(b)}$Without the ALEPH silicon calorimeter.

$^{(c)}$With the ALEPH silicon calorimeter.

$^{(d)}$This error has not been estimated accurately by the ALEPH collaboration since it is known to be small with respect to the statistical error.

	ALEPH	DELPHI	L3	OPAL
M_Z(GeV)	91.187±0.009	91.187±0.009	91.195±0.009	91.182±0.009
Γ_Z(GeV)	2.501±0.011	2.482±0.012	2.493±0.010	2.483±0.012
σ^0_h(nb)	41.61±0.16	41.02±0.27	41.33±0.26	41.71±0.23
R_e	20.58±0.15	20.70±0.18	20.90±0.16	20.83±0.16
R_μ	20.82±0.15	20.48±0.15	21.02±0.16	20.78±0.11
R_τ	20.62±0.17	20.88±0.20	20.78±0.20	21.01±0.15
$A^{0,e}_{FB}$	0.0185±0.0059	0.0237±0.0092	0.0135±0.0078	0.0062±0.0080
$A^{0,\mu}_{FB}$	0.0147±0.0047	0.0143±0.0050	0.0167±0.0064	0.0099±0.0042
$A^{0,\tau}_{FB}$	0.0182±0.0053	0.0213±0.0068	0.0257±0.0089	0.0205±0.0052

Table 3: Line shape and asymmetry parameters from 9-parameter fits to the data of the four LEP experiments.

$\mathcal{V}$:	ALEPH	DELPHI	L3	OPAL
ALEPH	$\sigma_A A \sigma_A^T$	$(C_E + C_2)^T$	$(C_E + C_1)^T$	$(C_E + C_2)^T$
DELPHI	$C_E + C_2$	$\sigma_D D \sigma_D^T$	$(C_E + C_2)^T$	$(C_E + C_3)^T$
L3	$C_E + C_1$	$C_E + C_2$	$\sigma_L L \sigma_L^T$	$(C_E + C_2)^T$
OPAL	$C_E + C_2$	$C_E + C_3$	$C_E + C_2$	$\sigma_O O \sigma_O^T$

Table 4: The covariance matrix $\mathcal{V}$ used for the LEP average of parameters. The parameter errors of the individual experiments (σ_A, σ_D, σ_L, σ_O) and the correlation matrices of the individual experiments(A, D, L, O, see Table 5) include the effect of experiment specific and common uncertainties. The submatrix C_E (see Table 6) accounts for common systematic uncertainties due to the LEP energy calibration. The submatrices C_1, C_2 and C_3 account for the common systematic error of the absolute normalization due to the theoretical uncertainty in the small angle Bhabha cross section. They are zero everywhere except for the diagonal element for σ_{h}^0. The theoretical errors quoted are assumed to be 100% correlated among the experiments and we use $C_1(\sigma_{\mathrm{h}}^0, \sigma_{\mathrm{h}}^0) = 0.0025 \cdot 0.0025 \cdot \sigma_{\mathrm{h}}^{0\,2}$, $C_2(\sigma_{\mathrm{h}}^0, \sigma_{\mathrm{h}}^0) = 0.0025 \cdot 0.0030 \cdot \sigma_{\mathrm{h}}^{0\,2}$ and $C_3(\sigma_{\mathrm{h}}^0, \sigma_{\mathrm{h}}^0) = 0.0030 \cdot 0.0030 \cdot \sigma_{\mathrm{h}}^{0\,2}$.

A:	M_Z	Γ_Z	σ_h^0	R_e	R_μ	R_τ	$A_{FB}^{0,e}$	$A_{FB}^{0,\mu}$	$A_{FB}^{0,\tau}$
M_Z	1.000	0.017	0.022	−0.003	0.002	−0.001	0.102	0.124	0.110
Γ_Z	0.017	1.000	−0.128	−0.001	0.016	−0.001	0.006	0.005	0.005
σ_h^0	0.022	−0.128	1.000	0.161	0.156	0.139	0.006	0.006	0.006
R_e	−0.003	−0.001	0.161	1.000	0.090	0.078	0.005	0.000	0.000
R_μ	0.002	0.016	0.156	0.090	1.000	0.076	0.001	0.009	0.000
R_τ	−0.001	−0.001	0.139	0.078	0.076	1.000	0.000	0.000	0.008
$A_{FB}^{0,e}$	0.102	0.006	0.006	0.005	0.001	0.000	1.000	0.050	0.047
$A_{FB}^{0,\mu}$	0.124	0.005	0.006	0.000	0.009	0.000	0.050	1.000	0.048
$A_{FB}^{0,\tau}$	0.110	0.005	0.006	0.000	0.000	0.008	0.047	0.048	1.000
D:	M_Z	Γ_Z	σ_h^0	R_e	R_μ	R_τ	$A_{FB}^{0,e}$	$A_{FB}^{0,\mu}$	$A_{FB}^{0,\tau}$
M_Z	1.000	−0.007	0.017	0.009	0.002	0.001	0.066	0.121	0.087
Γ_Z	−0.007	1.000	−0.161	−0.014	−0.016	−0.010	0.001	−0.004	−0.003
σ_h^0	0.017	−0.161	1.000	0.095	0.113	0.087	−0.002	−0.007	−0.006
R_e	0.009	−0.014	0.095	1.000	0.096	0.074	0.006	0.002	0.002
R_μ	0.002	−0.016	0.113	0.096	1.000	0.087	0.001	0.014	0.001
R_τ	0.001	−0.010	0.087	0.074	0.087	1.000	0.000	0.001	0.015
$A_{FB}^{0,e}$	0.066	0.001	−0.002	0.006	0.001	0.000	1.000	0.031	0.021
$A_{FB}^{0,\mu}$	0.121	−0.004	−0.007	0.002	0.014	0.001	0.031	1.000	0.040
$A_{FB}^{0,\tau}$	0.087	−0.003	−0.006	0.002	0.001	0.015	0.021	0.040	1.000
L:	M_Z	Γ_Z	σ_h^0	R_e	R_μ	R_τ	$A_{FB}^{0,e}$	$A_{FB}^{0,\mu}$	$A_{FB}^{0,\tau}$
M_Z	1.000	0.076	−0.004	0.009	0.001	0.000	0.042	0.051	0.036
Γ_Z	0.076	1.000	−0.105	−0.009	−0.005	−0.003	0.001	0.001	0.001
σ_h^0	−0.004	−0.105	1.000	−0.045	0.064	0.052	−0.010	−0.009	−0.006
R_e	0.009	−0.009	−0.045	1.000	0.058	0.045	0.013	0.002	0.001
R_μ	0.001	−0.005	0.064	0.058	1.000	0.046	0.000	0.011	0.000
R_τ	0.000	−0.003	0.052	0.045	0.046	1.000	0.000	0.000	0.010
$A_{FB}^{0,e}$	0.042	0.001	−0.010	0.013	0.000	0.000	1.000	0.020	0.015
$A_{FB}^{0,\mu}$	0.051	0.001	−0.009	0.002	0.011	0.000	0.020	1.000	0.017
$A_{FB}^{0,\tau}$	0.036	0.001	−0.006	0.001	0.000	0.010	0.015	0.017	1.000
O:	M_Z	Γ_Z	σ_h^0	R_e	R_μ	R_τ	$A_{FB}^{0,e}$	$A_{FB}^{0,\mu}$	$A_{FB}^{0,\tau}$
M_Z	1.000	0.003	−0.014	0.169	−0.008	−0.012	−0.081	0.080	0.077
Γ_Z	0.003	1.000	−0.089	0.022	0.035	0.012	−0.004	0.010	0.017
σ_h^0	−0.014	−0.089	1.000	0.042	0.195	0.147	0.087	−0.019	−0.013
R_e	0.169	0.022	0.042	1.000	0.135	0.079	−0.165	0.075	0.065
R_μ	−0.008	0.035	0.195	0.135	1.000	0.099	0.004	0.004	−0.003
R_τ	−0.012	0.012	0.147	0.079	0.099	1.000	0.008	−0.007	0.000
$A_{FB}^{0,e}$	−0.081	−0.004	0.087	−0.165	0.004	0.008	1.000	−0.040	−0.035
$A_{FB}^{0,\mu}$	0.080	0.010	−0.019	0.075	0.004	−0.007	−0.040	1.000	0.035
$A_{FB}^{0,\tau}$	0.077	0.017	−0.013	0.065	−0.003	0.000	−0.035	0.035	1.000

Table 5: The matrices A, D, L, and O used in the covariance matrix $\mathcal{V}$ (see definition in Table 4) for the average of the parameters of the 9 parameter fits from the individual LEP experiments.

C_E:	σ_E	C								
		M_Z	Γ_Z	σ_h^0	R_e	R_μ	R_τ	$A_{FB}^{0,e}$	$A_{FB}^{0,\mu}$	$A_{FB}^{0,\tau}$
M_Z(GeV)	6.3 MeV	1.	−0.38	0.	0.	0.	0.	0.	0.	0.
Γ_Z(GeV)	4.5 MeV	−0.38	1.	0.	0.	0.	0.	0.	0.	0.
σ_h^0(nb)	0	0.	0.	1.	0.	0.	0.	0.	0.	0.
R_e	0	0.	0.	0.	1.	0.	0.	0.	0.	0.
R_μ	0	0.	0.	0.	0.	1.	0.	0.	0.	0.
R_τ	0	0.	0.	0.	0.	0.	1.	0.	0.	0.
$A_{FB}^{0,e}$	0.001	0.	0.	0.	0.	0.	0.	1.	−1.	−1.
$A_{FB}^{0,\mu}$	0.001	0.	0.	0.	0.	0.	0.	−1.	1.	1.
$A_{FB}^{0,\tau}$	0.001	0.	0.	0.	0.	0.	0.	−1.	1.	1.

Table 6: The matrix $C_E = \sigma_E C \sigma_E^T$ used in the covariance matrix, $\mathcal{V}$ (see definition in Table 4) for the average of the parameters of the 9 parameter fits from the individual LEP experiments. Due to the t-channel contribution $A_{FB}^{0,e}$ has a slope with centre-of-mass energy which is of opposite sign to that of $A_{FB}^{0,\mu}$ and $A_{FB}^{0,\tau}$; hence the anticorrelation.

Parameter	Average Value
M_Z(GeV)	91.187±0.007
Γ_Z(GeV)	2.489±0.007
σ_h^0(nb)	41.56±0.14
R_e	20.743±0.080
R_μ	20.764±0.069
R_τ	20.832±0.088
$A_{FB}^{0,e}$	0.0153±0.0038
$A_{FB}^{0,\mu}$	0.0132±0.0026
$A_{FB}^{0,\tau}$	0.0204±0.0032

Table 7: Average line shape and asymmetry parameters from the data of the four LEP experiments given in Table 3, without the assumption of lepton universality. The $\chi^2/(d.o.f.)$ of the average is 27.1/27.

	M_Z	Γ_Z	σ_h^0	R_e	R_μ	R_τ	$A_{FB}^{0,e}$	$A_{FB}^{0,\mu}$	$A_{FB}^{0,\tau}$
M_Z	1.000	−0.157	0.006	0.029	−0.002	−0.003	0.025	0.056	0.048
Γ_Z	−0.157	1.000	−0.070	0.001	0.006	−0.001	0.000	0.004	0.006
σ_h^0	0.006	−0.070	1.000	0.071	0.100	0.085	0.008	−0.002	−0.001
R_e	0.029	0.001	0.071	1.000	0.102	0.072	−0.023	0.021	0.018
R_μ	−0.002	0.006	0.100	0.102	1.000	0.076	−0.001	0.007	−0.002
R_τ	−0.003	−0.001	0.085	0.072	0.076	1.000	0.000	−0.002	0.006
$A_{FB}^{0,e}$	0.025	0.000	0.008	−0.023	−0.001	0.000	1.000	−0.062	−0.048
$A_{FB}^{0,\mu}$	0.056	0.004	−0.002	0.021	0.007	−0.002	−0.062	1.000	0.119
$A_{FB}^{0,\tau}$	0.048	0.006	−0.001	0.018	−0.002	0.006	−0.048	0.119	1.000

Table 8: The correlation matrix for the set of parameters given in Table 7.

Parameter	Average Value
M_Z(GeV)	91.187±0.007
Γ_Z(GeV)	2.489±0.007
σ_h^0(nb)	41.56±0.14
R_ℓ	20.763±0.049
$A_{FB}^{0,\ell}$	0.0158±0.0018

Table 9: Average line shape and asymmetry parameters from the results of the four LEP experiments given in Table 3, assuming lepton universality. The $\chi^2/(d.o.f.)$ of the average is 30.8/31.

	M_Z	Γ_Z	σ_h^0	R_ℓ	$A_{FB}^{0,\ell}$
M_Z	1.000	−0.157	0.007	0.012	0.075
Γ_Z	−0.157	1.000	−0.070	0.003	0.006
σ_h^0	0.007	−0.070	1.000	0.137	0.003
R_ℓ	0.012	0.003	0.137	1.000	0.008
$A_{FB}^{0,\ell}$	0.075	0.006	0.003	0.008	1.000

Table 10: The correlation matrix for the set of parameters given in Table 9.

	ALEPH	DELPHI	L3	OPAL
M_Z(GeV)	91.187±0.009	91.187±0.009	91.195±0.009	91.181±0.009
Γ_Z(GeV)	2.501±0.011	2.482±0.012	2.493±0.010	2.482±0.012
σ_h^0(nb)	41.61±0.16	41.02±0.27	41.33±0.26	41.70±0.23
R_ℓ	20.68±0.10	20.65±0.11	20.92±0.11	20.835±0.086
$A_{FB}^{0,\ell}$	0.0168±0.0032	0.0179±0.0038	0.0178±0.0044	0.0128±0.0030

Table 11: Line shape and asymmetry parameters from 5-parameter fits to the data of the four LEP experiments, assuming lepton universality. As the definition of R_ℓ is slightly different among the experiments, the LEP averages quoted in this note are based on the results of the 9-parameter fits given in Table 3.

Without Lepton Universality:	
Γ_{ee}(MeV)	83.86±0.30
$\Gamma_{\mu\mu}$(MeV)	83.78±0.40
$\Gamma_{\tau\tau}$(MeV)	83.50±0.45
With Lepton Universality:	
Γ_l(MeV)	83.82±0.27
Γ_{had}(MeV)	1740.3±5.9
Γ_{inv}(MeV)	497.6±4.3
$g_{V_l}^2$	0.00134±0.00015
$g_{A_l}^2$	0.25088±0.00083
$\sin^2\theta_{eff}^{lept}$	0.2318±0.0010

Table 12: Average values of some derived parameters.

3 The τ Polarization

The τ polarization $\mathcal{P}_\tau$ is determined by measuring the longitudinal polarization of τ pairs produced in Z^0 decays. It is defined as

$$\mathcal{P}_\tau = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \tag{5}$$

where σ_R and σ_L are the cross-sections for the production of a right-handed and left-handed τ^-, respectively.

Correcting for small QED effects due to photon exchange and interference, and initial state radiation, the angular dependence of $\mathcal{P}_\tau$, as a function of the angle θ between the e^- and the τ^-, is given by:

$$\mathcal{P}_\tau(\cos\theta) = -\frac{\mathcal{A}_\tau + \mathcal{A}_e \frac{2\cos\theta}{1+\cos^2\theta}}{1 + \mathcal{A}_\tau \mathcal{A}_e \frac{2\cos\theta}{1+\cos^2\theta}}, \tag{6}$$

with $\mathcal{A}_e$ and $\mathcal{A}_\tau$ defined in eqn. (2). When averaged over all production angles $\mathcal{P}_\tau$ is a measurement of $\mathcal{A}_\tau$, while as a function of $\cos\theta$, $\mathcal{P}_\tau(\cos\theta)$ provides nearly independent determinations of $\mathcal{A}_\tau$ and $\mathcal{A}_e$, allowing thus a test of the universality of the couplings of the Z^0 to e and τ.

Tables 13 and 14 show the results for $\mathcal{A}_\tau$ and $\mathcal{A}_e$ respectively obtained by the four experiments [9, 10, 11, 12] and their combination. Common systematic uncertainties among the experiments are expected to be much smaller than the present error of the average and have been neglected.

ALEPH	('90 + '91)	0.143 ± 0.023
DELPHI	('90 + '91 + '92), prel.	0.151 ± 0.029
L3	('90 + '91 + '92), prel.	0.133 ± 0.024
OPAL	('90 + '91), prel.	0.117 ± 0.046
LEP Average		0.139 ± 0.014

Table 13: LEP results for $\mathcal{A}_\tau$. For L3 and OPAL $\mathcal{A}_\tau$ is derived from the measured values quoted.

ALEPH	('90 + '91)	0.120 ± 0.026
OPAL	('90 + '91), prel.	0.231 ± 0.083
LEP Average		0.130 ± 0.025

Table 14: LEP results for $\mathcal{A}_e$.

The partial widths of the Z^0 into leptons, the lepton forward-backward asymmetries, the τ polarization and the τ polarization asymmetry can all be combined to determine the vector and axial vector couplings for e, μ and τ. The asymmetries determine the ratio g_{V_ℓ}/g_{A_ℓ} (see eqn. (1)) while the axial vector coupling squared is derived from the leptonic partial width (see eqn. (3)). The corresponding results for the effective lepton couplings are given in Table 15. Figure 2 shows the one standard deviation contours in the g_{A_ℓ}-g_{V_ℓ} plane. The measured ratios of the e, μ and τ couplings provide a test of universality:

$$g_{A_\mu}/g_{A_e} = 1.0006 \pm 0.0026\ , \quad g_{A_\tau}/g_{A_e} = 0.9990 \pm 0.0029\ ,$$

$$g_{V_\mu}/g_{V_e} = 0.77 \pm 0.21\ , \quad g_{V_\tau}/g_{V_e} = 1.00 \pm 0.13\ .$$

g_{V_e}	-0.0373 ± 0.0031
g_{V_μ}	-0.0288 ± 0.0064
g_{V_τ}	-0.0372 ± 0.0032
g_{A_e}	-0.50096 ± 0.00093
g_{A_μ}	-0.5013 ± 0.0012
g_{A_τ}	-0.5005 ± 0.0014
g_{V_ℓ}	-0.0359 ± 0.0018
g_{A_ℓ}	-0.50093 ± 0.00082

Table 15: Results for the leptonic vector and axial vector couplings without and with the assumption of lepton universality.

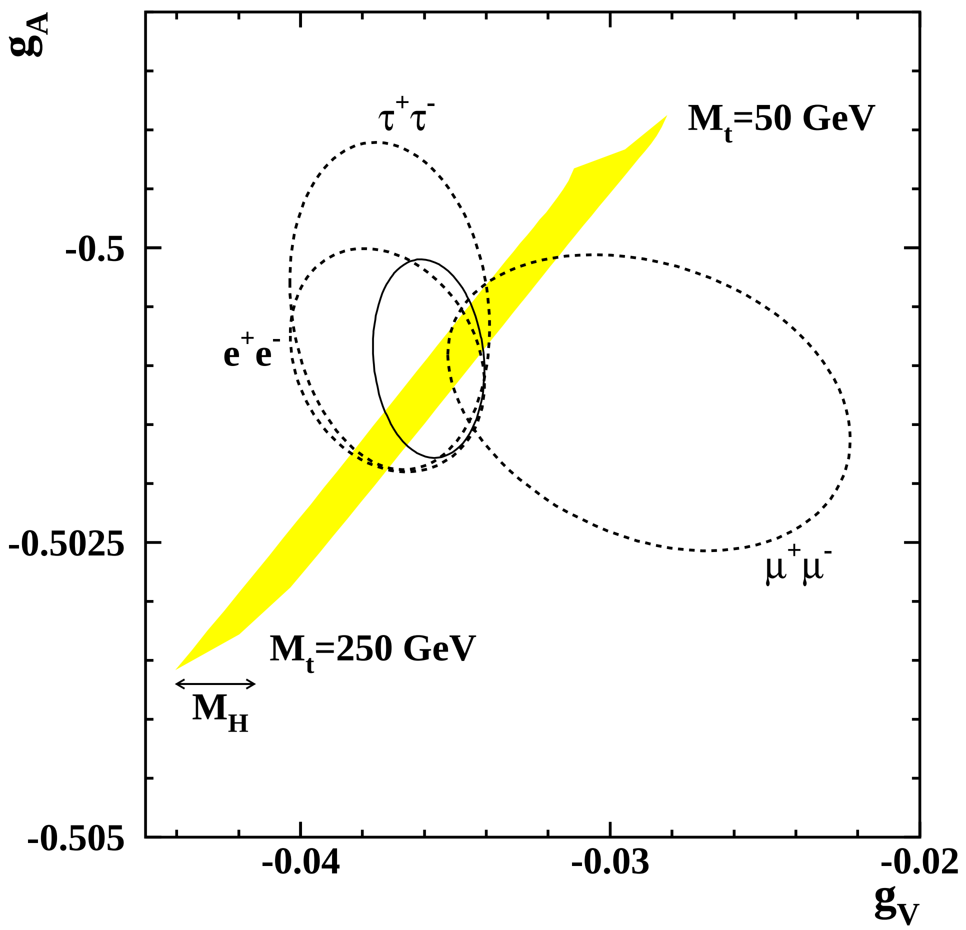

Figure 2: One standard deviation contours (39% probability) in the g_{V_ℓ}-g_{A_ℓ} plane. The solid contour results from a fit assuming lepton universality. The shaded band represents the Standard Model prediction.

4 Electroweak Results with b and c Quarks

The averages which follow represent a first attempt by the LEP Electroweak Working Group to combine measurements of $\Gamma_{b\bar{b}}$, $\Gamma_{c\bar{c}}$, $A_{FB}^{b\bar{b}}$ and $A_{FB}^{c\bar{c}}$. Many different analysis techniques are used and different parameters are determined simultaneously by the experiments. Furthermore, the individual experiments choose slightly different values of certain input parameters in order to determine the central values of the quoted measurements or different ranges of these parameters in assessing the systematic errors. A typical example is the treatment of $\Gamma_{c\bar{c}}/\Gamma_{had}$, which is either determined in a simultaneous fit with $\Gamma_{b\bar{b}}/\Gamma_{had}$, or its value is taken from the Standard Model prediction or an independent experimental measurement. Similar problems apply to non Standard Model inputs, such as the choice of semileptonic decay models. In the future the experiments will work together to define the necessary standards for the quotation of heavy flavour results to allow for a consistent choice of input assumptions in the averaging procedure. In the present analysis, the selection of input parameters has remained the choice of the individual experiments. Some cross checks on the sensitivity of the averaging procedure, in particular to the treatment of correlated errors, were made and are discussed below.

4.1 Measurement of $\Gamma_{b\bar{b}}/\Gamma_{had}$

The measurements of $\Gamma_{b\bar{b}}/\Gamma_{had}$ from the individual LEP experiments are given in Table 16. The measurements are divided into three categories, based on lepton tagging, event shapes and lifetime tagging. For each of the three groups, the systematic errors were divided into those specific to an experiment ("Uncorr. sys.") and those in common between the results ("Common sys."). The evaluation of the common systematic errors involves some approximations, because of the different assumptions on the input parameters discussed above. For the average values, where three separate errors are quoted, the first is statistical, the second due to uncorrelated systematic errors and the third due to errors considered to be common. In the following sections, the common systematic errors for each category and the choice of weights for each measurement are discussed. A slightly more precise average could have been obtained by adopting the approach used in [25], in which the total error is minimized taking into account all correlations. At the present accuracy, however, the methods used here to assign the weight to each measurement should be an adequate approximation.

4.1.1 Measurements of $\Gamma_{b\bar{b}}/\Gamma_{had}$ using lepton tagging

The sources of common systematic errors for the lepton tagging methods are:

- Semileptonic decay model
- Semileptonic branching ratios of b and c hadrons.
- Contributions from $c\bar{c}$ events, in particular $\Gamma_{c\bar{c}}/\Gamma_{had}$

The relative importance of these errors depends on the method. For example, using samples of events with one or two tagged leptons it is possible to fit for several parameters including $\Gamma_{b\bar{b}}/\Gamma_{had}$, $\Gamma_{c\bar{c}}/\Gamma_{had}$, semileptonic branching ratios and the average b fragmentation parameter. Even in a double tag method where these parameters are not explicitly extracted, their effect on the b tagging efficiency is derived directly from the data. Each measurement was weighted according to its total error in forming the average.

$\Gamma_{b\bar{b}}/\Gamma_{had}$ from lepton tags				
	ALEPH 90–92 [13] prel.	DELPHI 91–92 [14] prel.	L3 90–91 [15] prel.	OPAL[(a)] 90–91 [16],[17] prel.
$\Gamma_{b\bar{b}}/\Gamma_{had}$	0.2223	0.222	0.2184	0.221
Stat. error	0.0042	0.009	0.0081	0.004
Uncorr. sys.	0.0033	0.002	0.0045	0.008
Common sys.	0.0046	0.004	0.0059	0.006
LEP average	$0.221 \pm 0.003 \pm 0.002 \pm 0.005$ ($\chi^2/(d.o.f.) = 0.1/3$)			

$\Gamma_{b\bar{b}}/\Gamma_{had}$ from event shapes			
	ALEPH 90–91 [18]	DELPHI 90–91 [19]	L3 91 [20]
$\Gamma_{b\bar{b}}/\Gamma_{had}$	0.228	0.232	0.222
Stat. error	0.005	0.005	0.003
Uncorr. sys.	0.005	0.011	0.002
Common sys.	0.001	0.013	0.006
LEP average	$0.226 \pm 0.002 \pm 0.003 \pm 0.004$ ($\chi^2/(d.o.f.) = 1.1/2$)		

$\Gamma_{b\bar{b}}/\Gamma_{had}$ from lifetime tags					
	ALEPH 92 (1) [21]	DELPHI 91–92 (2) [22] prel.	DELPHI 91 (3) [23] prel.	OPAL 90 (1) [24]	OPAL 91-92 (4) [25] prel.
$\Gamma_{b\bar{b}}/\Gamma_{had}$	0.2192	0.206	0.222	0.222	0.2133
Stat. error	0.0022	0.0066	0.007	0.007	0.0041
Uncorr. sys.	0.0020	0.0037	0.0063	0.008	0.0027
Common sys.	0.0023	0.0047	0.003	0.002[(b)]	0.0033
LEP average	$0.2169 \pm 0.0018 \pm 0.0015 \pm 0.0027$ ($\chi^2/(d.o.f.) = 3.6/4$)				

$\Gamma_{b\bar{b}}/\Gamma_{had}$ combining leptons, event shapes and lifetimes	
LEP average	0.2200 ± 0.0027 ($\chi^2/(d.o.f.) = 5.6/11$)

Table 16: The various measurements of $\Gamma_{b\bar{b}}/\Gamma_{had}$ from the individual LEP experiments and the LEP averages. Where three errors are quoted for the LEP averages, the first is statistical, the second from uncorrelated systematic errors and the third from common systematic error.
[(a)] Combination of the published single tag [16] and preliminary double tag [17] results, taking into account the common errors.
[(b)] Estimated by using the LEP average value of $\Gamma_{c\bar{c}}/\Gamma_{had}$ in the expression provided in [24].

4.1.2 Measurements of $\Gamma_{b\bar{b}}/\Gamma_{had}$ using event shapes

The Boosted Sphericity Product results were neglected because the overall errors from this method, and in particular the systematic uncertainty due to the light quarks, are very large.

The ALEPH result uses a double tag method, where the b tagging efficiency is measured directly from the data, while the DELPHI and L3 results are based on a single tag method with more input from Monte Carlo simulation. The systematic errors arising from varying Monte Carlo parameters including b fragmentation were treated as common among the experiments. Each measurement was weighted according to its total error in forming the average.

4.1.3 Measurements of $\Gamma_{b\bar{b}}/\Gamma_{had}$ using lifetime tagging

The lifetime tagging methods used are: (1) Impact parameter double tagging, (2) Decay length and lepton mixed double tag, (3) Decay length plus event shape and (4) Decay length double tag.

The two main contributions to the common systematic errors are:

- Hemisphere correlations. All the systematic errors quoted by the experiments have been taken to be fully correlated.
- $\Gamma_{c\bar{c}}/\Gamma_{had}$. The range quoted by each experiment is used, and the resulting errors are assumed to be fully correlated. The double decay length plus event shape method is unusual in that the event shape tag removes a large fraction of charm events, leading to a small error from this source.

Clearly other sources of correlation exist, for example due to charm multiplicity, the charmed hadron composition, b and c hadron production in uds events, and K^0 production. These effects are smaller and neglecting these sources of correlation is expected to compensate partly the pessimistic choice of full correlation from $\Gamma_{c\bar{c}}/\Gamma_{had}$ and hemisphere correlations. For the average value the weight for each measurement was computed from the statistical and uncorrelated systematic errors only.

4.1.4 Overall LEP average measurement of $\Gamma_{b\bar{b}}/\Gamma_{had}$

To compute the overall average of all the LEP measurements, the results from the three tagging categories were assumed to be entirely uncorrelated. This is not exact, as there are statistical correlations between the results using different methods by the same experiment (for example the mixed lepton/lifetime result of DELPHI). In addition, there are some sources of common systematic error, for example due to $\Gamma_{c\bar{c}}/\Gamma_{had}$ and b quark fragmentation. The different measurements are sensitive to different aspects of the fragmentation process, and the correlations are difficult to extract.

$$\Gamma_{b\bar{b}}/\Gamma_{had}(\text{LEP Average}) = 0.2200 \pm 0.0027.$$

As a cross check of the sensitivity to the assumption that the results from the three categories are uncorrelated, the $\Gamma_{c\bar{c}}/\Gamma_{had}$ dependence of each measurement (which was estimated when it was not explicitly available) was used to derive an error correlated among the three categories. This resulted in a small shift in the central value of $\Gamma_{b\bar{b}}/\Gamma_{had}$, with the same error as quoted above.

The overall average is more sensitive to the method used to assign weights to the individual measurements. The effect of choosing the weights by minimizing the total error, taking into account correlations, resulted in a positive shift in $\Gamma_{b\bar{b}}/\Gamma_{had}$ of about 25% of the error quoted above. The change in the error was not significant. The dominant effect came from changing the relative weights of the lifetime measurements.

4.2 Measurement of $\Gamma_{c\bar{c}}/\Gamma_{had}$

The values for $\Gamma_{c\bar{c}}/\Gamma_{had}$ are unchanged since the Montreal conference. The input numbers are given in Table 17. The LEP average value quoted is a simple weighted average assuming no common errors. Taking a "worst case" scenario where the full systematic error for the three measurements using a D^* tag are correlated, the result becomes $\Gamma_{c\bar{c}}/\Gamma_{had} = 0.172 \pm 0.015$, a negligible shift from the result using the simple weights.

$\Gamma_{c\bar{c}}/\Gamma_{had}$	
ALEPH ℓ 90-91 [26]	$0.165 \pm 0.005 \pm 0.019$
DELPHI lifetime+shape 91 [19]	$0.151 \pm 0.008 \pm 0.041$
DELPHI D^* inclusive 89–May 90 [27]	$0.162 \pm 0.030 \pm 0.050$
DELPHI D^* + lifetime 91 [28]	$0.187 \pm 0.031 \pm 0.023$
OPAL D^* 90-91 [29]	$0.188 \pm 0.015 \pm 0.026$
LEP average	0.171 ± 0.014

Table 17: The measurements of $\Gamma_{c\bar{c}}/\Gamma_{had}$ from the individual LEP experiments, with statistical and systematic errors, and the LEP average.

4.3 Measurements of $A_{FB}^{b\bar{b}}$ and $A_{FB}^{c\bar{c}}$

Asymmetry measurements using a lepton tag, a lifetime tag in combination with a jet charge measurement and a D^* tag are available. The individual values are given in Table 18. The quoted values of $A_{FB}^{b\bar{b}}$ are all corrected for the effect of $B^0\overline{B^0}$ mixing. The quoted values of $A_{FB}^{b\bar{b}}$ based on a lepton tag and the ALEPH measurement using a vertex tag have all been corrected using the same LEP average value for the mixing parameter, $\chi = 0.115 \pm 0.009 \pm 0.006$ [30, 32, 39]. This value was derived from lepton tag measurements only, and the second error is due to the semileptonic decay model uncertainty. Apart from the correction for mixing, no other corrections have been made to $A_{FB}^{b\bar{b}}$ or $A_{FB}^{c\bar{c}}$, in particular no QCD corrections. As with the heavy flavour partial widths, there are some discrepancies among the asymmetry measurements in the choice of input parameters. At the present time, however, the contribution to the error from effects such as the choice of the semileptonic decay model is still small compared to the total error, so that it is meaningful to form an average.

The systematic uncertainties in $A_{FB}^{b\bar{b}}$ have been divided into an uncorrelated part (due for example to backgrounds, lepton identification uncertainties and detector effects), and a correlated part, to which the following effects contribute:

- semileptonic decay model (SL model),
- semileptonic branching ratio (SL BR),
- fragmentation,
- $\Gamma_{b\bar{b}}$ and $\Gamma_{c\bar{c}}$,
- $B^0\overline{B^0}$ mixing,
- the $c\bar{c}$ forward-backward asymmetry, $A_{FB}^{c\bar{c}}$.

	ALEPH ℓ, high p_t 90–92 [30] prel.	DELPHI μ, high p_t 91–92 [31] prel.	L3 ℓ 90–92 [32] prel.	OPAL ℓ 90–91 [33]	ALEPH vertex 92 [34] prel.	DELPHI vertex 91–92 [35] prel.	LEP Average
$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(\%)$	8.1	10.2	9.1	9.1	10.9	11.6	9.4
Stat. error	±1.0	±1.6	±1.0	±1.8	±1.2	±1.9	±0.5
Uncorr. sys.	±0.1	±0.9	±0.5	±0.4	±0.5	±1.7	±0.2
SL model	±0.12	(b)	(b)	±0.30	(d)	(d)	
SL BR	±0.09	±0.46	±0.18	±0.25	(d)	(d)	
Fragmentation	±0.12	±0.46	±0.08	±0.05	±0.10	±0.80	
$\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{c\bar{c}}}$	±0.08	(b)	±0.04	±0.45	±0.11	±0.50	
Mixing	±0.23	±0.30	±0.26	±0.26	∓0.13	±0.30	
$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$	(c)	±0.46	(a)	(a)	(c)	(e)	
Total corr.	±0.3	±0.8	±0.3	±0.6	±0.2	±1.0	±0.3
$\sqrt{s}$ (GeV)	91.27	91.27	91.27	91.23	91.28	91.27	91.27

	ALEPH ℓ 90–91 [26]	L3 ℓ 90–92 [32] prel.	OPAL ℓ 90–91 [33]	ALEPH D* 90–91 [36] prel.	DELPHI D* 90–91 [37] prel.	OPAL D* 90–92 [38]	LEP Average
$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(\%)$	9.9	6.0	1.4	6.8	10.7	5.2	6.6
Stat. error	±2.0	±2.2	±3.0	±4.2	±7.5	±2.8	±1.2
Uncorr. sys.	±1.5	±2.2	±1.7	±0.7	±1.0	±0.9	±0.7
SL model	±0.76	(b)	±0.62	(d)	(d)	(d)	
SL BR	±0.51	±1.31	±0.66	(d)	(d)	(d)	
Fragmentation	(a)	±0.45	±0.21	±0.18	±0.13(f)	±0.12	
$\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{c\bar{c}}}$	(a)	±0.22	±0.60	(a)	(f)	(a)	
Mixing, $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$	(d,a)	(d,a)	(d,a)	±0.54	±0.86(f)	±0.80	
D* BR	(d)	(d)	(d)	±0.18	±0.13(f)	±0.12	
Total corr.	±0.9	±1.4	±1.1	±0.6	±0.9	±0.8	±0.7
$\sqrt{s}$ (GeV)	91.23	91.27	91.23	91.23	91.23	91.27	91.25

Table 18: The measurements of $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$, corrected for mixing, and $A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$ from the individual LEP experiments and the LEP average. For each measurement the statistical and uncorrelated systematic errors are given, followed by the correlated errors broken into 6 subcategories and the total correlated error.
(a) Fitted simultaneously and included in the statistical or uncorrelated error,
(b) Treated together with SL BR error,
(c) Standard Model relationship of $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ and $A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$ assumed,
(d) Not applicable,
(e) Negligible,
(f) Breakdown of systematic error inferred from other D* tag measurements.

The uncertainties of individual measurements in each of these 6 categories are given in Table 18, where the various reasons for a zero contribution are also explained. The covariance matrix, $\mathcal{V}$, was constructed assuming that the systematic uncertainties in each category were fully correlated between the measurements, with the exception of the mixing uncertainty in the ALEPH vertex tag measurement, which has been shown to have a negative correlation with the mixing uncertainty in the lepton tag measurements. The weight for each measurement was then chosen so as to minimize the $\chi^2 = \Delta^T \mathcal{V}^{-1} \Delta$, where Δ denotes the vector of residuals of the combined value to the individual results. The combined result is

$$A_{FB}^{b\overline{b}} = (9.4 \pm 0.5 \pm 0.2 \pm 0.3)\% \qquad (\chi^2/(d.o.f.) = 3.8/5).$$

The three quoted errors are the statistical, that due to uncorrelated systematic errors and that due to common systematic uncertainties respectively.

For the measurements of $A_{FB}^{c\overline{c}}$, the covariance matrix was calculated as before but dividing the correlated errors into 6 slightly different categories:

- semileptonic decay model,
- semileptonic branching ratios,
- fragmentation,
- $\Gamma_{b\overline{b}}$ and $\Gamma_{c\overline{c}}$,
- $B^0\overline{B^0}$ mixing and $A_{FB}^{b\overline{b}}$,
- Probability to produce a D^* in $b\overline{b}$ and $c\overline{c}$ events and D^* branching ratios (D^* BR).

In this case the average result, with the quoted errors divided up as before, is:

$$A_{FB}^{c\overline{c}} = (6.6 \pm 1.2 \pm 0.7 \pm 0.7)\% \qquad (\chi^2/(d.o.f.) = 4.5/5).$$

Several of the measurements evaluate $A_{FB}^{b\overline{b}}$ and $A_{FB}^{c\overline{c}}$ simultaneously. It was therefore verified that extracting the two average asymmetries simultaneously gave consistent results. The full covariance matrix was constructed, taking into account correlations between the two asymmetries in an analogous way to the combined OPAL measurements using the lepton and D^* tags [38]. The correlation between $A_{FB}^{b\overline{b}}$ and $A_{FB}^{c\overline{c}}$ was observed to be between 5% and 25%, depending on the assumptions made on the correlations due to each source of systematic error.

ALEPH measures the asymmetries and mixing simultaneously using leptons. This has the advantage that the semileptonic decay modelling is taken into account for $A_{FB}^{b\overline{b}}$ and the mixing simultaneously, leading to a small reduction in the systematic error. It has been verified that taking this into account leads to no significant difference in the LEP average at this stage. It was also checked that the averages were not sensitive to imposing an additional error due to the semileptonic decay model for the measurements where this was relevant but not evaluated separately. The possible additional error was estimated from the quoted errors.

	$\Delta A_{FB}^{b\overline{b}}(\%)$	$\Delta A_{FB}^{c\overline{c}}(\%)$
QED	$+0.4$	$+1.0$
QCD	$+0.3 \pm 0.1$	$+0.2 \pm 0.1$
DE	-0.2	-0.3
total	$+0.5$	$+0.9$

Table 19: Corrections to be made to the $b\overline{b}$ and $c\overline{c}$ forward-backward asymmetries in order to derive pole asymmetries, as explained in the text.

In order to derive the pole asymmetries, $A_{FB}^{0,b}$ and $A_{FB}^{0,c}$ (defined in an analogous way to those for leptons in eqn. (1)), the corrections given in Table 19 should be applied. The QED shift is mainly due to initial state radiation. The QCD correction is calculated using a k-factor [40] of 0.75 ± 0.25. This uncertainty reflects the range of explicit or implicit event selection criteria of the experimental methods which biases the relative fractions of 2- and 3-jet events. The DE correction is due to the difference between M_Z and the average centre of mass energy of the measurements. The pole asymmetries are therefore $A_{FB}^{0,b} = 0.099 \pm 0.006$ and $A_{FB}^{0,c} = 0.075 \pm 0.015$ for b and c quarks, respectively.

4.4 Measurements of $A_{FB}^{b\bar{b}}$ and $A_{FB}^{c\bar{c}}$ off peak

The off-peak values for $A_{FB}^{b\bar{b}}$ and $A_{FB}^{c\bar{c}}$ are all based on 1990 and 1991 data. The two quoted errors are statistical and systematic, respectively. The LEP average $A_{FB}^{b\bar{b}}$ numbers are the result of weighting by the total error. The quoted systematic on this average is a conservative estimate of the common systematic error. The same weights are used to find the average $\sqrt{s}$ values.

	$A_{FB}^{b\bar{b}}$	$\sqrt{s}$ (GeV)
ALEPH ℓ	$0.027 \pm 0.039 \pm 0.005$	89.4
	$0.102 \pm 0.037 \pm 0.009$	92.8
L3 ℓ	$0.025 \pm 0.051 \pm 0.007$	89.67
	$0.062 \pm 0.042 \pm 0.007$	92.81
OPAL ℓ	$0.071 \pm 0.054 \pm 0.007$	89.66
	$0.131 \pm 0.047 \pm 0.013$	92.75
LEP Average	$0.037 \pm 0.027 \pm 0.005$	89.6
	$0.096 \pm 0.024 \pm 0.007$	92.8
	$A_{FB}^{c\bar{c}}$	$\sqrt{s}$ (GeV)
OPAL D*	$-0.14 \pm 0.14 \pm 0.03$	89.35
	$0.18 \pm 0.12 \pm 0.03$	92.85

Table 20: The measurements of $A_{FB}^{b\bar{b}}$ and $A_{FB}^{c\bar{c}}$ off peak from the individual LEP experiments and the LEP averages. In each case the errors are statistical and systematic.

5 The Hadronic Charge Asymmetry

ALEPH [41, 44], DELPHI [42], and OPAL [43] have provided measurements of the charge asymmetry in the inclusive hadronic event sample. Within the $SU(2) \times U(1)$ structure the quark asymmetries are determined by one unique value of the effective mixing angle, $\sin^2\theta_{eff}^{lept}$, up to fermion dependent vertex corrections which are small and assigned to their Standard Model values. There are different analysis techniques. ALEPH and DELPHI use an integrated charge average, OPAL uses a weight function method and DELPHI presents, as a second method, a charge assignment on an event by event basis using different combinations of tracks. The experimental values quoted for the average forward-backward charge asymmetry, $\langle Q_{FB} \rangle$, cannot be directly compared as some of them include detector dependent effects, acceptances and detector efficiencies. We therefore use $\sin^2\theta_{eff}^{lept}$ as a means of combining the results summarized in Table 21.

The dominant systematic error comes from the modelling of the charge flow in the fragmentation process. The JETSET fragmentation is used by all experiments as reference, the HERWIG model is used for comparison. The JETSET parameters are varied to estimate the systematic errors. The strange quark production from string breaking (the s/u parameter) is the most sensitive parameter. The central values chosen by the experiments for these parameters are, however, not the same. The fragmentation errors are, for the moment, considered fully correlated among the experiments. The present average is not very sensitive to the treatment of common uncertainties.

The determination of $\sin^2\theta_{eff}^{\rm lept}$ is not fully consistent between experiments. Possible ambiguities due to QCD corrections may cause a change of at most 0.0007 on $\sin^2\theta_{eff}^{\rm lept}$, still below the fragmentation and experimental errors.

Exp.	data	s/u	$\sin^2\theta_{eff}^{\rm lept}$
ALEPH	89-92 prel.	0.315 ± 0.045	$0.2317 \pm 0.0013 \pm 0.0011$
DELPHI	90-91	0.315 ± 0.045	$0.2345 \pm 0.0030 \pm 0.0027$
OPAL	90-91	0.285 ± 0.050	$0.2321 \pm 0.0017 \pm 0.0028$
Average			$0.2320 \pm 0.0011 \pm 0.0011$

Table 21: Summary of the determination of $\sin^2\theta_{eff}^{\rm lept}$ from inclusive hadronic charge asymmetries at LEP.

6 The Effective Electroweak Mixing Angle $\sin^2\theta_{eff}^{\rm lept}$

Several electroweak measurements from LEP can be combined into a single observable, the effective electroweak mixing angle $\sin^2\theta_{eff}^{\rm lept}$ defined in eqn. (4), without making any strong model specific assumptions. For a combined average of $\sin^2\theta_{eff}^{\rm lept}$ from $A_{\rm FB}^{0,\ell}$, $\mathcal{A}_\tau$ and $\mathcal{A}_{\rm e}$ only the assumption of lepton universality, already inherent in the definition given in eqn. (4), is needed. In practice no further assumption is involved if the quark forward-backward asymmetries $A_{\rm FB}^{\rm b\bar{b}}$ and $A_{\rm FB}^{\rm c\bar{c}}$ are included in this average, as these asymmetries have a reduced sensitivity to corrections particular to the hadronic vertex. The results of these determinations of $\sin^2\theta_{eff}^{\rm lept}$ and their combination are shown in Table 22.

	$\sin^2\theta_{eff}^{\rm lept}$
$A_{\rm FB}^{0,\ell}$	0.2318 ± 0.0010
$\mathcal{A}_\tau$	0.2325 ± 0.0018
$\mathcal{A}_{\rm e}$	0.2337 ± 0.0032
$A_{\rm FB}^{0,\rm b}$	0.2322 ± 0.0011
$A_{\rm FB}^{0,\rm c}$	0.2313 ± 0.0036
Average	0.2321 ± 0.0007

Table 22: Comparison of several direct determinations of $\sin^2\theta_{eff}^{\rm lept}$ from asymmetries. The average is obtained as a weighted average assuming no correlations. The $\chi^2/(d.o.f.)$ of the average is 0.5/4.

The determination of $\sin^2\theta_{eff}^{\rm lept}$ from the hadronic charge asymmetry, $\langle {\rm Q_{FB}} \rangle$, involves additional assumptions as $\langle {\rm Q_{FB}} \rangle$ depends on the fractional composition of the primary quark flavours in the hadronic sample. Combining the result $\sin^2\theta_{eff}^{\rm lept} = 0.2320 \pm 0.0016$ from $\langle {\rm Q_{FB}} \rangle$ with the average of the measurements in Table 22 we obtain $\sin^2\theta_{eff}^{\rm lept} = 0.2321 \pm 0.0006$.

7 Standard Model Constraints

The precision measurements collected in the preceeding sections can be used to check the validity of the Standard Model and to infer valuable information about its basic parameters. Their accuracy makes them sensitive to the top quark mass, M_{t}, and the mass of the Higgs boson, M_{H}, through the loop corrections. The leading top quark dependence is quadratic and allows a determination of M_{t}. The main dependence on M_{H} is logarithmic and, with the present data accuracy, it is not expected to allow a meaningful determination of M_{H}.

The measurements collected above, are summarized in Table 23 and presented in Figure 3 together with their Standard Model prediction as a function of M_{t}. The bands in the Standard Model predic-

measurement	result
a) LEP	
line-shape and lepton asymmetries:	
M_{Z}	91.187 ± 0.007 GeV
Γ_{Z}	2.489 ± 0.007 GeV
σ_{h}^{0}	41.56 ± 0.14 nb
R_{ℓ}	20.763 ± 0.049
$A_{\mathrm{FB}}^{0,\ell}$	0.0158 ± 0.0018
+ correlation matrix (Table 9)	
τ polarization asymmetries:	
$\mathcal{A}_{\tau}$	0.139 ± 0.014
$\mathcal{A}_{\mathrm{e}}$	0.130 ± 0.025
b and c quark results:	
$A_{\mathrm{FB}}^{0,\mathrm{b}}$	0.099 ± 0.006
$A_{\mathrm{FB}}^{0,\mathrm{c}}$	0.075 ± 0.015
$\Gamma_{\mathrm{b\overline{b}}}/\Gamma_{\mathrm{had}}$	0.2200 ± 0.0027
$\mathrm{q\overline{q}}$ charge asymmetry:	
$\sin^2\theta_{eff}^{\mathrm{lept}}$ from $\langle \mathrm{Q_{FB}} \rangle$	0.2320 ± 0.0016
b) $\mathrm{p\overline{p}}$ and νN	
$M_{\mathrm{W}}/M_{\mathrm{Z}}$ (UA2)	0.8813 ± 0.0041
M_{W} (CDF)	79.91 ± 0.39
$1 - M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2(\nu N)$	0.2256 ± 0.0047

Table 23: Summary of measurements included in the combined analysis of Standard Model parameters. Section a) summarizes LEP averages, section b) electroweak precision tests from hadron colliders and νN-scattering.

tions reflect the expected variation of each quantity due to a variation of the strong coupling constant $\alpha_s(M_{\mathrm{Z}}^2) = 0.123 \pm 0.006$ [45] and M_{H} in the interval $60 \leq M_{\mathrm{H}}$ [GeV] ≤ 1000 for $M_{\mathrm{Z}} = 91.187$ GeV.

Table 24 shows the constraints obtained on M_{t} and $\alpha_s(M_{\mathrm{Z}}^2)$ when fitting all the above measurements to the most up to date Standard Model calculations [8]. Since the data are not expected to have a sensitivity with respect to M_{H}, the fits have been repeated for $M_{\mathrm{H}} = 60$, 300 and 1000 GeV and the difference in the fitted parameters is quoted as the uncertainty. We present the results obtained using only the LEP data and also the results obtained using in addition the measurements of $M_{\rm W}$ and $M_{\rm W}/M_{\rm Z}$ from CDF [47] and UA2 [46], and the measurements of the neutrino neutral to charged current ratios from CDHS [48], CHARM [49] and CCFR [50]. The χ^2 of the fits shows that the agreement of the data with their Standard Model predictions is excellent. The value of $\alpha_s(M_{\rm Z}^2)$ resulting from these fits is in very good agreement with the value obtained from event shape measurements at LEP ($\alpha_s(M_{\rm Z}^2) = 0.123 \pm 0.006$ [45]) and of similar precision.

	LEP	LEP + Collider and ν data
$M_{\rm t}$ (GeV)	$166^{+17}_{-19}\,^{+19}_{-22}$	$164^{+16}_{-17}\,^{+18}_{-21}$
$\alpha_s(M_{\rm Z}^2)$	$0.120 \pm 0.006\ \pm 0.002$	$0.120 \pm 0.006\ \pm 0.002$
$\chi^2/(d.o.f.)$	3.5/8	4.4/11
$\sin^2\theta_{eff}^{\rm lept}$	$0.2324 \pm 0.0005\ ^{+0.0001}_{-0.0002}$	$0.2325 \pm 0.0005\ ^{+0.0001}_{-0.0002}$
$1 - M_{\rm W}^2/M_{\rm Z}^2$	$0.2255 \pm 0.0019\ ^{+0.0005}_{-0.0003}$	$0.2257 \pm 0.0017\ ^{+0.0004}_{-0.0003}$
$M_{\rm W}$ (GeV)	$80.25 \pm 0.10\ ^{+0.02}_{-0.03}$	$80.24 \pm 0.09\ ^{+0.01}_{-0.02}$

Table 24: Results of fits to LEP and other data for $M_{\rm t}$ and $\alpha_s(M_{\rm Z}^2)$. No external constraint on $\alpha_s(M_{\rm Z}^2)$ has been imposed. In the third column also the data from the $\rm p\overline{p}$ experiments UA2 [46]: $M_{\rm W}/M_{\rm Z} = 0.8813 \pm 0.0041$, and CDF [47]: $M_{\rm W} = 79.91 \pm 0.39$ GeV and from the neutrino experiments, CDHS [48], CHARM [49] and CCFR [50]: $1 - M_{\rm W}^2/M_{\rm Z}^2 = 0.2256 \pm 0.0047$ are included. The central values and the first errors quoted refer to $M_{\rm H} = 300$ GeV. The second errors correspond to the variation of the central value when varying $M_{\rm H}$ in the interval 60 GeV $< M_{\rm H} <$ 1000 GeV.

The theoretical uncertainties in the electroweak parameter predictions have been properly propagated in the fits. The dominant ones are the uncertainties coming from the light quark contribution to the vacuum polarizations and secondly the uncertainty coming from the scale choice in the heavy quark loops. The technical uncertainty has been estimated by comparing the results obtained with different Electroweak Libraries. The effect of missing higher orders has been estimated by comparing resummation prescriptions which treat in a different way subleading terms. These two latter uncertainties have been estimated to be well below the one coming from the light quark contribution to loops for all the observables under consideration. The magnitude of the two dominant theoretical uncertainties has been taken from [51]. Their effect can be judged from the fact that if they would not have been taken into account, the results of Table 24 would have changed as follows: a positive shift of about 2 GeV in the top mass and a reduction of 1 to 2 GeV in its uncertainty. Therefore, at present their effect in the top mass determination is small, but will be significant for the anticipated accuracy of future LEP results.

The fact that the $\Gamma_{\rm b\overline{b}}/\Gamma_{\rm had}$ measurement has an accuracy of the order of one per cent, allows the estimation of the top mass entering the $\rm Z^0b\overline{b}$ vertex correction in a way which is largely free from assumptions on the Higgs sector structure. Figure 4 shows the χ^2 curves as a function of $M_{\rm t}$ obtained by fitting this measurement alone. This curve has been obtained by propagating the uncertainties in $M_{\rm Z}$ and $\alpha_s(M_{\rm Z}^2)$ as well as the theoretical uncertainties mentioned above, together with a variation of the b-quark mass in the range $m_{\rm b} = 4.7 \pm 0.2$ GeV. The combined effect of the above uncertainties is small. As can be seen from Figure 3b), the measurement of $\Gamma_{\rm b\overline{b}}/\Gamma_{\rm had}$ is consistent with the Standard Model prediction. The fact that the χ^2 at the minimum is not zero shows that there is no $M_{\rm t}$ value which in the Standard Model would lead to the measured central value of $\Gamma_{\rm b\overline{b}}/\Gamma_{\rm had} = 0.2200$. This is due to the fact that the measurement would prefer an $M_{\rm t}^2$ correction of different sign than the one predicted by the Standard Model. Therefore, the fast increase of χ^2 as a function of M_t is somewhat 'artificial' and its interpretation has to be taken with care.

Figure 5 shows, for the fit in Table 24 column 3, the χ^2 value as a function of M_t for the three values of M_H considered in Table 24. These curves demonstrate that the data impose stringent constraints on the mass of the up to now elusive top quark. With the direct determination of M_t and future improvements in the accuracy of LEP results we may also hope to obtain interesting constraints on M_H.

At present, the increase in χ^2 when M_H is changed between the two extreme values is about 2.0, which does not allow the derivation of any meaningful constraints on M_H. In addition, an increase of 1.4 out of 2.0 is traced back to the contribution of the $\Gamma_{b\bar{b}}/\Gamma_{had}$ measurement. As stated above, the measurement of $\Gamma_{b\bar{b}}/\Gamma_{had}$ 'artificially' constrains M_t. The rest of the data give a determination of M_t which is strongly correlated with the assumed value of M_H used as input to the fit. Therefore the inclusion of the $\Gamma_{b\bar{b}}/\Gamma_{had}$ measurement also artificially constrains M_H. Our conclusion is that the present increase in χ^2 with M_H is 'artificial' and hence its interpretation has to be taken with care.

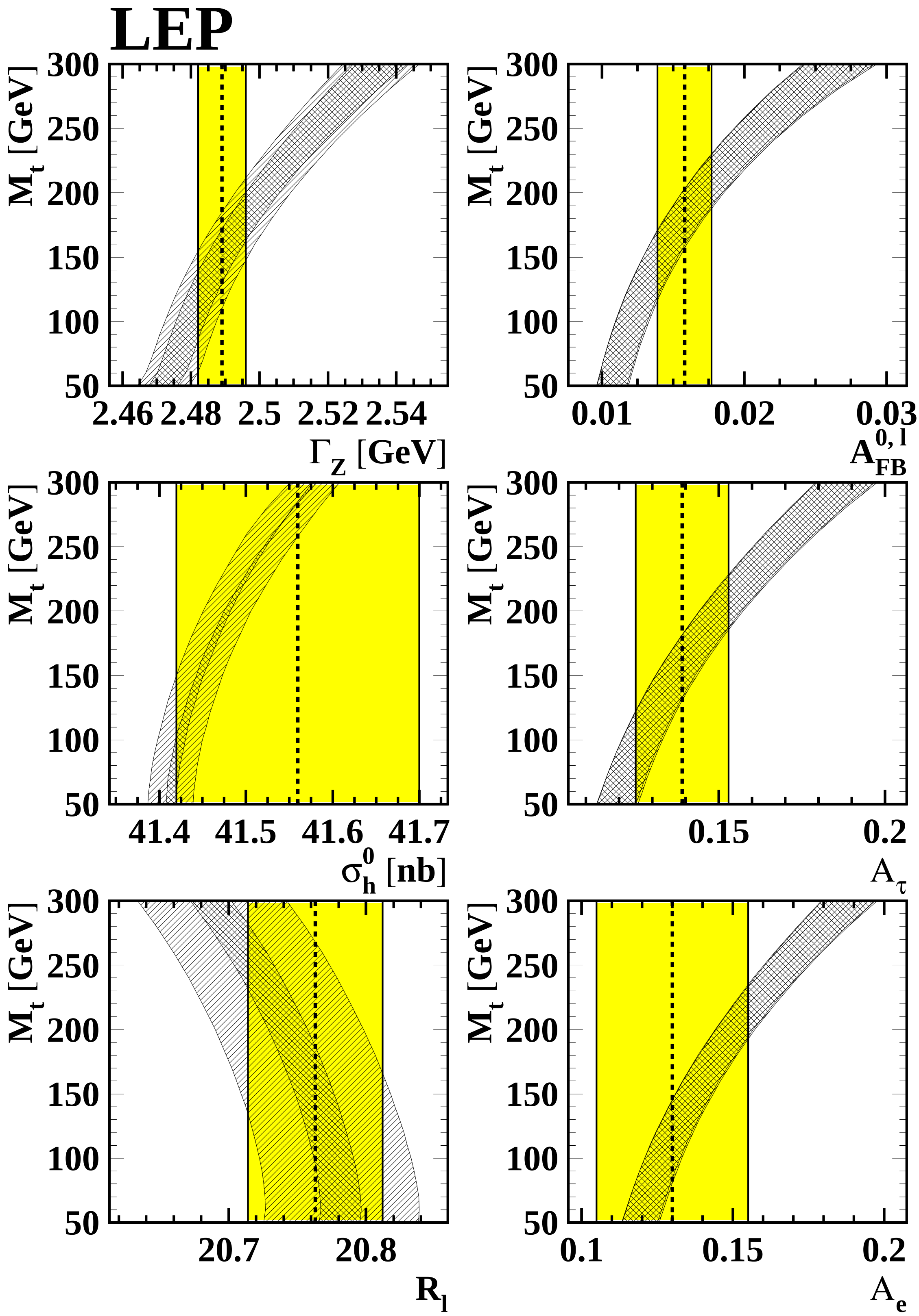

Figure 3: a) Comparison of LEP measurements with the Standard Model prediction as a function of M_t. The cross-hatched area shows the variation of the Standard Model prediction with M_H spanning the interval $60 < M_H\,(\mathrm{GeV}) < 1000$ and the singly-hatched area corresponds to a variation of $\alpha_s(M_Z^2)$ within the interval $\alpha_s(M_Z^2) = 0.123 \pm 0.006$. The experimental errors on the parameters are indicated as vertical bands.

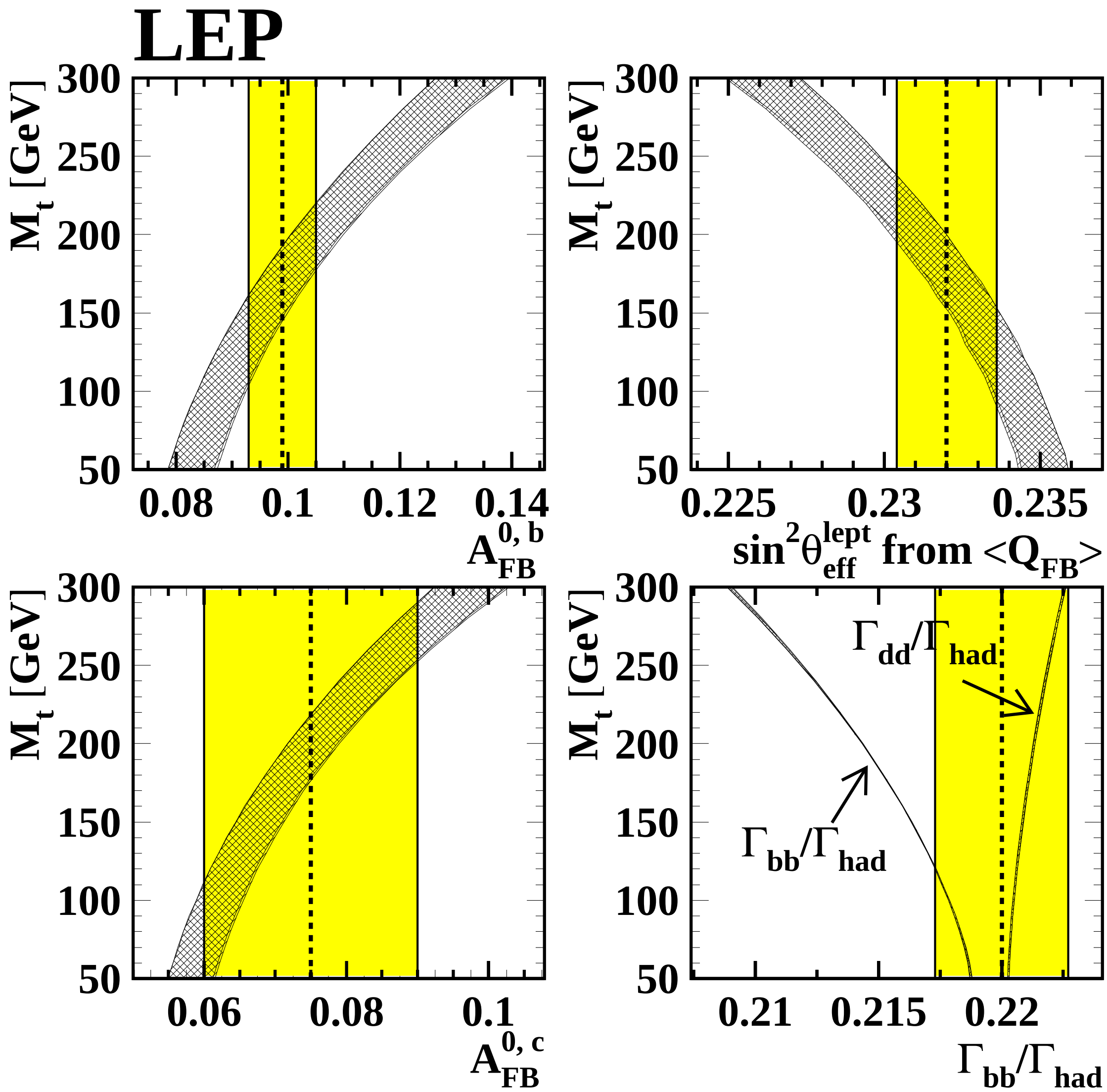

Figure 3: b) Comparison of LEP measurements with the Standard Model prediction as a function of M_t. The cross-hatched area shows the variation of the Standard Model prediction with M_H spanning the interval $60 < M_H\,(\mathrm{GeV}) < 1000$ and the singly-hatched area corresponds to a variation of $\alpha_s(M_Z^2)$ within the interval $\alpha_s(M_Z^2) = 0.123 \pm 0.006$. For the ratios of partial widths, $\Gamma_{b\bar{b}}/\Gamma_{had}$ and $\Gamma_{d\bar{d}}/\Gamma_{had}$, this variation nearly cancels. The experimental errors on the parameters are indicated as vertical bands.

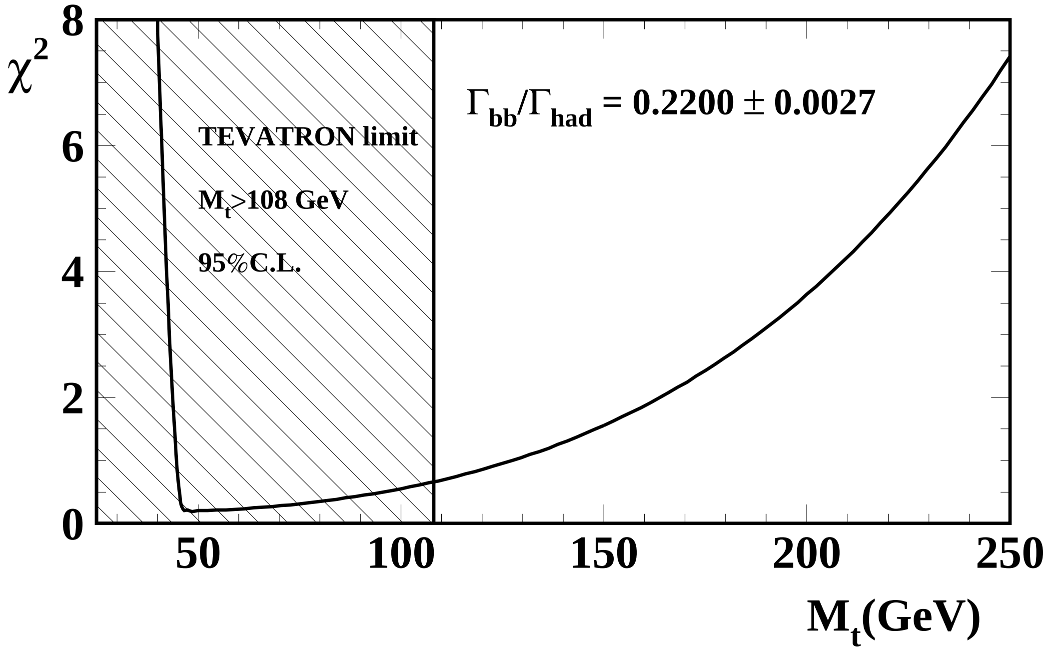

Figure 4: The χ^2 curve for the Standard Model fit to $\Gamma_{b\bar{b}}$ alone. M_Z and α_s have been varied as free parameters constrained by their input values given in Table 23. The χ^2 curve is not sensitive to the value of M_H when varied in the interval $60 < M_H\,(\mathrm{GeV}) < 1000$.

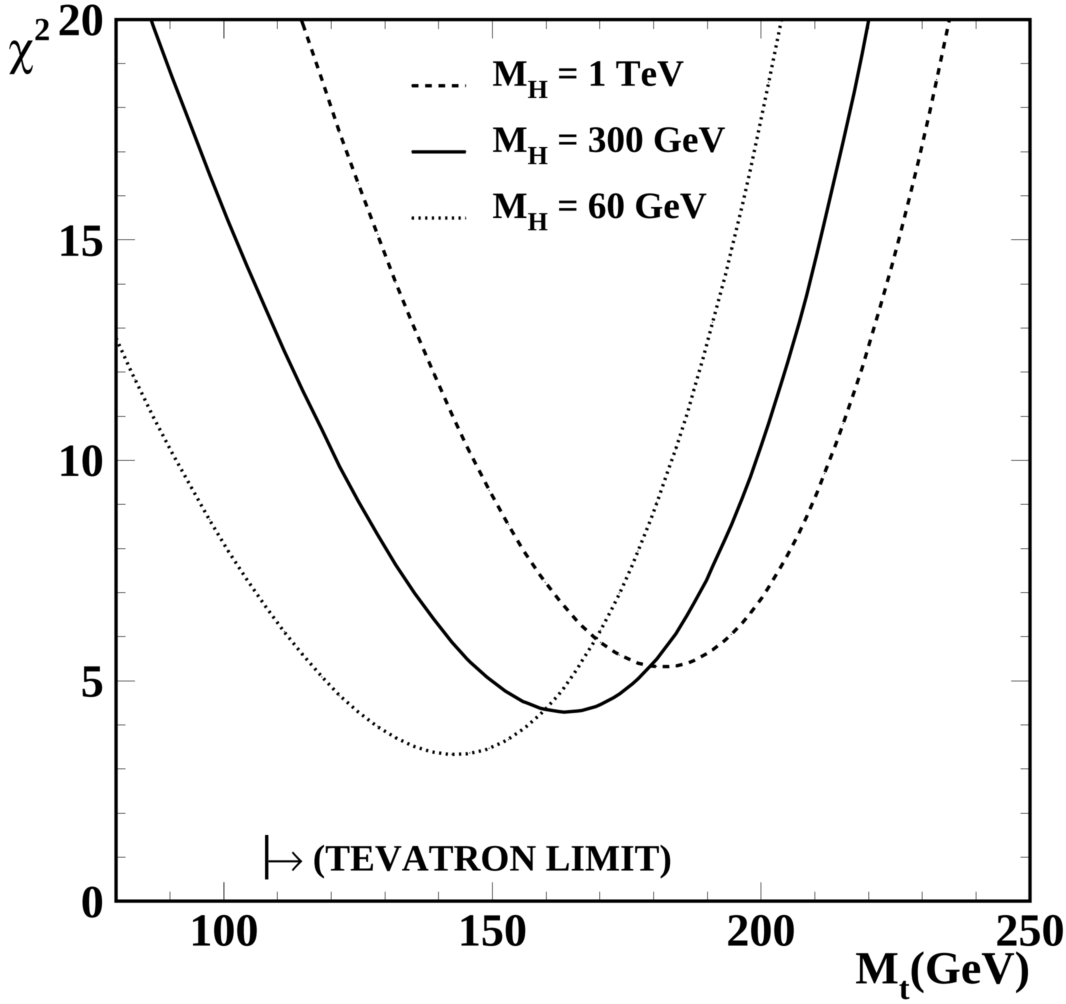

Figure 5: The χ^2 curves for the Standard Model fit in Table 24, column 3 to the electroweak precision measurements listed in Table 23 as a function of M_t for three different Higgs mass values spanning the interval $60 < M_H\,(\mathrm{GeV}) < 1000$.

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